# Globally Optimal Horizontal Condenser Design

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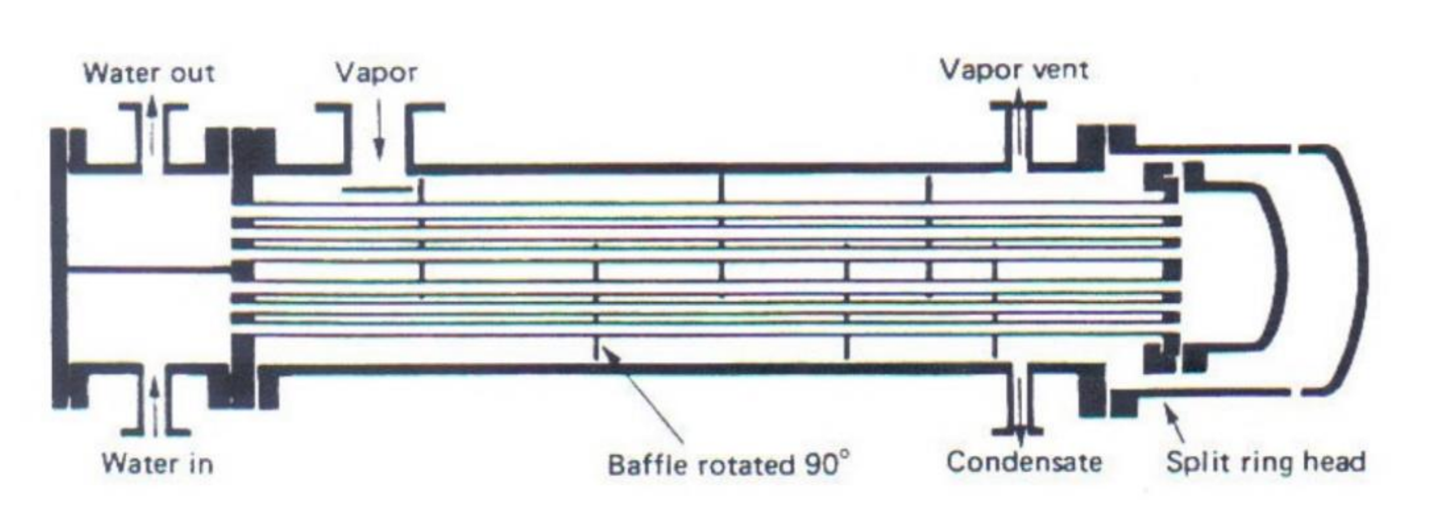
- **Introduction**

This project aims at analyzing and comparing the application of two different computational approaches for optimal condenser design

- Mixed Integer Linear Prorgamming (MILP)
- Set-Trimming.

The models for both methods are compared and the diferences in performance are discussed.

- **Condenser Model**
  - ✓ The analysis is focused on shell and tube heat exchangers.
  - ✓ A horizontal single shell E-shell type is used.
  - ✓ An even number of tube passes is considered
  - ✓ Shell side condensation is assumed




Source: *Heat Exchanger Design Handbook*

# MILP

## Mixed-Integer Linear Programming

- The original problem equations are non-linear.
- Geometry is described using discrete variables.
- Model reformulation is performed to obtain a Linear Model.

# Set-Trimming

## Sequential Set Trimming

- Gradual elimination of infeasible subsets of candidate solutions is performed.
- This is done apllying sequentially diferente constraints of the problem
- When finished, the optimum is obtained by inspection, enumeration or mathematical programming.

# Workshop: New Tools for the Optimal Basic Design of Chemical Process Equipment and Flowsheets

PPGEQ programa de pós-graduação em engenharia química

UERJ Universidade do Estado do Rio de Janeiro

## MINLP

***Objective function:*** $Min \quad A$

Heat transfer area $A = Ntt \ \pi \ dte \ L$ Excess area $A \geq \left(1 + \frac{\widehat{Aexc}}{100}\right) * Areq$

***Heat Transfer Rate Equations*** $\hat{Q} = UAreq \ \widehat{\Delta Tlm} \ F$ $\widehat{\Delta Tlm} = \frac{(\widehat{Thi}-\widehat{Tco})-(\widehat{Tho}-\widehat{Tci})}{\ln\left(\frac{(\widehat{Thi}-\widehat{Tco})}{(\widehat{Tho}-\widehat{Tci})}\right)}$

$$F = \frac{(\hat{R}^2+1)^{0.5}\ln\left(\frac{(1-\hat{P})}{(1-\hat{R}\hat{P})}\right)}{(\hat{R}-1)\ln\left(\frac{2-\hat{P}\left(\hat{R}+1-(\hat{R}^2+1)^{0.5}\right)}{2-\hat{P}\left(\hat{R}+1+(\hat{R}^2+1)^{0.5}\right)}\right)} \quad \hat{R} = \frac{\widehat{Thi}-\widehat{Tho}}{\widehat{Tco}-\widehat{Tci}} \quad \hat{P} = \frac{\widehat{Tco}-\widehat{Tci}}{\widehat{Thi}-\widehat{Tci}}$$

***Shell-Side Thermal and Hydraulic Equations***

Shell heat transfer coeff. $hs = 0{,}954 \cdot \left[\frac{\widehat{\rho s}\,(\widehat{\rho s}-\widehat{\rho v s})\hat{g}\widehat{Ks}^3 \ L \ Ntt}{\widehat{ms}\widehat{\mu s}}\right]^{\frac{1}{3}} \cdot (N_{vert})^{-\frac{1}{6}}$

The total number of tubes $Ntt = Ntp \cdot Npt$ Tubes per vertical row $N_{vert} = \frac{0{,}78 \ Ds}{ltp^{vert}}$

The vertical tube pitch $ltp^{vert} = ltp \begin{cases} 1, \text{ if Square or Triangular pattern} \\ \frac{1}{\sqrt{2}}, \text{ if Rotated Square pattern} \\ \frac{1}{2}, \text{ if Rotated Triangular pattern} \end{cases}$

***Tube-Side Thermal and Hydraulic Equations***

Velocity in tubes $vt = \frac{4 \ \widehat{mt}}{Ntp \ \pi \ \widehat{\rho t} \ dti^2}$ Nusselt # $Nut = 0.023 \ Ret^{0.8} \widehat{Prt}^n$

Head loss tube-side $\frac{\Delta Pt}{\widehat{\rho t} \ \hat{g}} = \frac{ft \ Npt \ L \ vt^2}{2 \ \hat{g} \ dti} + \frac{K \ Npt \ vt^2}{2 \ \hat{g}}$ Darcy friction factor $ft = 0.014 + \frac{1.056}{Ret^{0.42}}$

***Overall Heat Transfer Coefficient:*** $U = \frac{1}{\frac{dte}{dti \ ht} + \frac{\widehat{Rft} \ dte}{dti} + \frac{dte \ln\left(\frac{dte}{dti}\right)}{2 \ \widehat{ktube}} + \widehat{Rfs} + \frac{1}{hs}}$

## MILP

***Objective function:*** $Min \quad A$

Heat transfer area $A = \pi \sum_{srow=1}^{srowmax} \widehat{pNtt}_{srow} \ \widehat{pdte}_{srow} \ \widehat{pL}_{srow} yrow_{srow}$ Excess area $A \geq \left(1 + \frac{\widehat{Aexc}}{100}\right) * Areq$

***Heat Transfer Rate Equations*** $\hat{Q} = UAreq \ \widehat{\Delta Tlm}\hat{F}_{srow}$ $\widehat{\Delta Tlm} = \frac{(\widehat{Thi}-\widehat{Tco})-(\widehat{Tho}-\widehat{Tci})}{\ln\left(\frac{(\widehat{Thi}-\widehat{Tco})}{(\widehat{Tho}-\widehat{Tci})}\right)}$

$$\hat{F}_{srow} = \frac{(\hat{R}^2+1)^{0.5}\ln\left(\frac{(1-\hat{P})}{(1-\hat{R}\hat{P})}\right)}{(\hat{R}-1)\ln\left(\frac{2-\hat{P}\left(\hat{R}+1-(\hat{R}^2+1)^{0.5}\right)}{2-\hat{P}\left(\hat{R}+1+(\hat{R}^2+1)^{0.5}\right)}\right)} \quad \hat{R} = \frac{\widehat{Thi}-\widehat{Tho}}{\widehat{Tco}-\widehat{Tci}} \quad \hat{P} = \frac{\widehat{Tco}-\widehat{Tci}}{\widehat{Thi}-\widehat{Tci}}$$

***Shell-Side Thermal and Hydraulic Equations***

Shell heat transfer coeff. $\widehat{phs}_{srow} = 0{,}994 \cdot \left[\frac{\widehat{\rho s}\,(\widehat{\rho s}-\widehat{\rho v s})\hat{g}\widehat{Ks}^3\widehat{pL}_{srow}\widehat{pNtt}_{srow}}{\widehat{ms}\widehat{\mu s}}\right]^{\frac{1}{3}} \cdot \left(\frac{\widehat{pDs}_{srow}}{\widehat{pproj}_{srow}\widehat{prp}_{srow}\widehat{pdte}_{srow}}\right)^{-\frac{1}{6}}$

The total number of tubes $Ntp = \sum_{srow=1}^{srowmax} \frac{\widehat{pNtt}_{srow}}{\widehat{pNpt}_{srow}} yrow_{srow}$ Tubes per vertical row $N_{vert} = \frac{0{,}78 \ Ds}{ltp^{vert}}$

The vertical tube pitch $ltp^{vert} = ltp \begin{cases} 1, \text{ if Square or Triangular pattern} \\ \frac{1}{\sqrt{2}}, \text{ if Rotated Square pattern} \\ \frac{1}{2}, \text{ if Rotated Triangular pattern} \end{cases}$

***Tube-Side Thermal and Hydraulic Equations***

Velocity in tubes $vt = \frac{4 \ \widehat{mt}}{\pi \ \widehat{\rho t}} \sum_{srow=1}^{srowmax} \frac{\widehat{pNpt}_{srow}}{\widehat{pNtt}_{srow}\widehat{pdti}_{srow}^{\,2}} yrow_{srow}$

Nusselt # $Nut = 0{,}023 \left(\frac{4 \ \widehat{mt}}{\pi \ \widehat{\mu t}}\right)^{0{,}8} \widehat{Prt}^n \sum_{srow=1}^{srowmax} \left(\frac{\widehat{pNpt}_{srow}}{\widehat{pNtt}_{srow}\widehat{pdti}_{srow}}\right)^{0{,}8} yrow_{srow}$

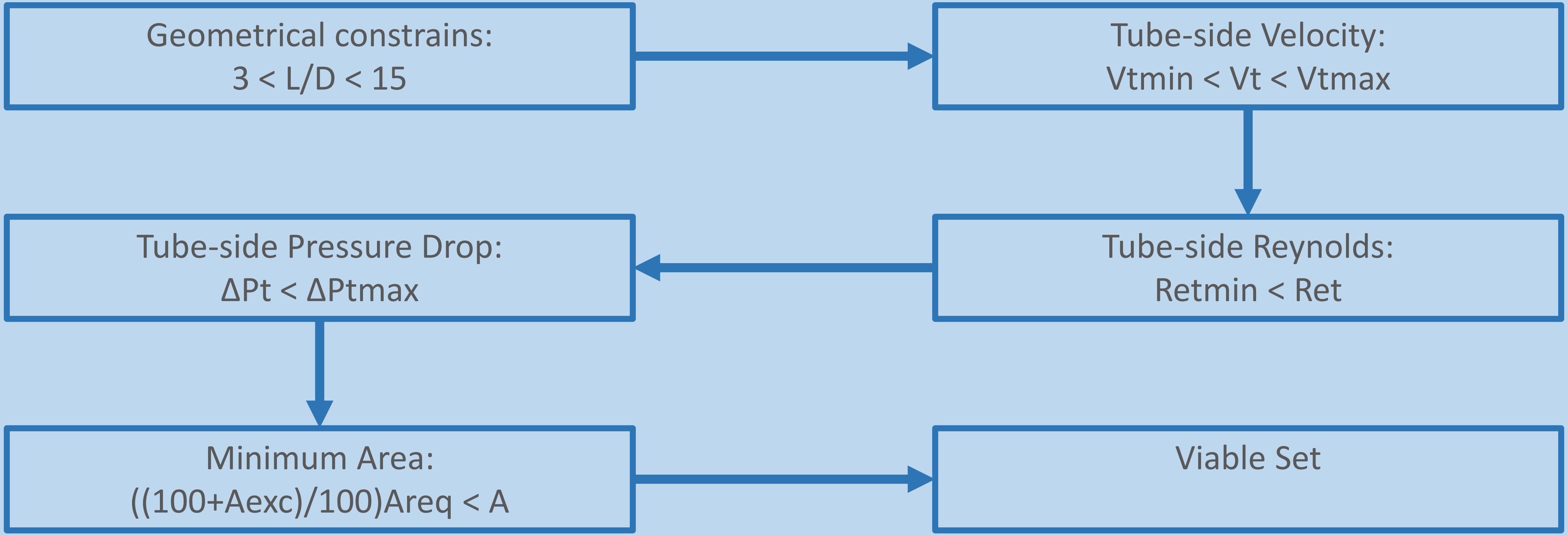
Head loss tube-side $\Delta Pt = \sum_{srow=1}^{srowmax} (p\widehat{\Delta Pturb1}_{srow} + p\widehat{\Delta Pturb2}_{srow} + (p\widehat{\Delta Ptcab}_{srow}\hat{K}_{srow})) \ yrow_{srow}$

Darcy friction factor $ft = 0.014 + \frac{1.056}{Ret^{0.42}}$

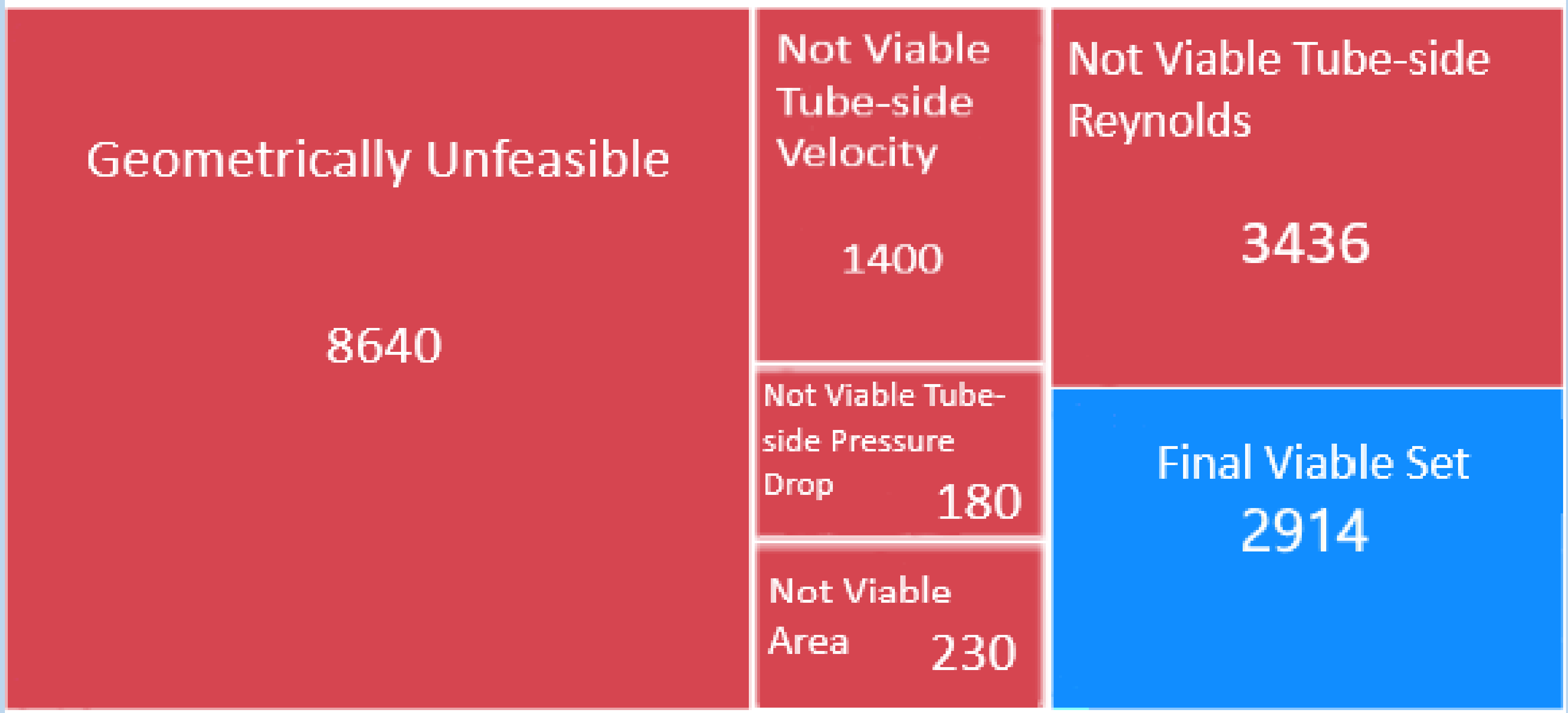
***Overall Heat Transfer Coefficient:***

$$\boldsymbol{U} = \frac{1}{\sum_{srow=1}^{srowmax} \frac{\widehat{pdte}_{srow} yrow_{srow}}{\widehat{pht}_{srow}\widehat{pdti}_{srow}} + \widehat{Rft} \sum_{srow=1}^{srowmax} \frac{\widehat{pdte}_{srow} yrow_{srow}}{\widehat{pdti}_{srow}} + \frac{\sum_{srow=1}^{srowmax} \widehat{pdte}_{srow} \ln\left(\frac{\widehat{pdte}_{srow}}{\widehat{pdti}_{srow}}\right) yrow_{srow}}{2 \ \widehat{Ktube}} + \widehat{Rfs} + \sum_{srow=1}^{srowmax} \frac{yrow_{srow}}{\widehat{phs}_{srow}}}$$

- **Set Trimming Procedure**



- **Set Trimming Procedure**



- **Examples**

**Table A1. Heat Exchanger Examples**

| Example | 1 | 2 | 3 |
|---|---|---|---|
| Service | Hot Water condenser | Methanol condenser | Acetone condenser |
| Hot stream | Hot Water | Methanol | Acetone |
| Cold stream | Cooling water | Cooling water | Cooling water |
| Tube-side stream | Cold | Cold | Cold |

- **Elapsed Time**

## Performance Comparison

| | Heat transfer area ($m^2$) | | Solution time (s) | |
|---|---|---|---|---|
| Example | MILP | Set Trimming | MILP | Set Trimming |
| 1 | 69.70 | 69.70 | 0.756 | 0.081 |
| 2 | 93.42 | 93.42 | 0.849 | 0.071 |
| 3 | 128.09 | 128.09 | 0.833 | 0.083 |

- **Conclusions**

  - A rigorous linear condenser model was presented
  - Set Trimming was also applied to the original nonlinear model.
  - We show that set trimming has superior computational time performance
    It is 10 TIMES FASTER